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FREQUENCIES, INPUT ADMITTANCES AND BANDWIDTHS OF THE NATURAL BENDING EIGENMODES IN XYLOPHONE BARS

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The results are reported of experimental measurements of the response of xylophone bars to a random signal applied at their centre and at one end. Empirical values are obtained with these spectra for the frequencies, input admittances and bandwidths corresponding to the natural bending eigenmodes of xylophone bars. The results explain the acoustical behaviour of the vibrating elements of xylophones and show that it is possible to estimate the vibrational effects of the geometrical shapes of such bars.

1. INTRODUCTION

A major characteristic of xylophonic instruments is that they have wooden bars as vibrating elements; these are graduated in length or width. This family of musical instruments includes the xylophones, other similar instruments with lower frequency ranges (marimbas) anc certain intermediate instruments such as the xylo-marimbas, etc. Xylophonic instruments also include devices of similar shape but are composed of metallic bars instead of wooden ones [1, 2] (glockenspiel, vibraphone, etc.). Despite this fairly broad range of possibilities, the types of instruments to be studied here are only those with wooden bars.

These musical instruments produce sounds with a well-defined pitch quality and can therefore be tuned to a specific scale, with a bar for each musical sound or note. Xylophonic bars are made of palissandre (rosewood), a kind of wood of high density and stiffness (density $\rho = 1050$ kg/m^3, Young's modulus along the grain direction $E = 19$ Gpa on average). They do not have a constant cross section, but rather have an undercut shape similar to that of a parabolic cylinder. They are attached to the instrument by means of two tensioned strings, which pass through two holes on each side of the bar.

Xylophonic instruments also have resonating elements, consisting of metallic tubes located immediately below each bar. The end of the tube farthest from the bar is closed, while that closest to the bar is open. The lengths of the tubes are graduated to ensure a fundamental frequency equal to the corresponding bar.

Xylophone bars are excited by a mallet, which in most cases is used to hit them in their central zone. Such mallets resemble small hammers with a spherical head, and can be made

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from a full range of materials of different hardnesses. This broad variety of mallets affords the possibility of achieving a large range of timbre effects [3].

There is some disagreement about the classification of xylophonic instruments. The reason for this is that there are no criteria for assessing the musical range of these instruments. It is therefore sometimes quite difficult to make a clear distinction between certain xylophones and their lower toned counterparts, the marimbas [1]. Consequently, to avoid confusion, the instrument analyzed here will be referred to as ‘‘xylophonic instrument‘‘, meaning only that it is an instrument with wooden bars as vibrating elements.

Within this general description of xylophonic instruments, their most remarkable acoustical characterisic can be said to be their ability to produce well-defined pitch sounds. These are generated by the flexural vibrations of the xylophonic bars when excited by the mallets. This is why undercut bars are used instead of others the shape of which is easier to obtain, such as constant section bars, because the latter do not exhibit a periodic flexural vibration.

The achievement of harmonic relationships between frequencies or a dominant periodicity with the undercutting effect has been studied in depth. In this sense, the studies undertaken by Rossing [1, 2] merit comment. This author developed a classification of a large variety of percussion instruments, relating the 1:4 integer relationship to low tone xylophonic instruments, such as the marimba, and the 1:3 ratio to treble tone instruments.

Bork [4, 5], Bork and Meyer [5] and also Moore [6] have also published very interesting experimental results in which the achievement of integer relationships between frequencies in bars is related to several kinds of change in their shape. These works are of great interest to instrument makers.

Finally, Orduña-Bustamante [7] conducted a study in which measurements of the acoustical effects of precise undercut shapes are compared with calculations from theoretical models including rotary inertia and shear stress.

The results to be presented here are those of an experimental analysis of the physico-acoustical function of xylophonic bars. Three kinds of vibrational parameters corresponding to natural flexural or bending eigenmodes are studied: frequencies, input admittances and bandwidths. In terms of these parameters the periodic oscillations of xylophonic bars, their patterns of vibration and the natural damping of their resonances can be classified.

2. EXPERIMENTAL SET-UP

In this work a method initially developed at the Department of Speech Communication and Music Acoustics of the K.T.H. at Stockholm [8] was used. A scheme of the experimental set-up used in this work can be seen in Figure 1. Essentially, it is composed

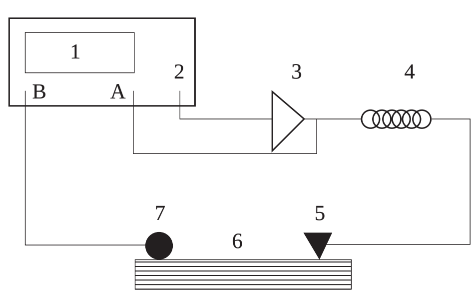


Figure 1. The experimental set-up for the vibrational property measurements. 1, Dual Channel Analyzer; 2, generator; 3, amplifier; 4, coil; 5, magnet; 6, mechanical system; 7, accelerometer; A and B, analyzer channels.

of a Dual Channel Signal Analyzer (Brüel & Kjaer Model 2034) which has a random signal generator with constant amplitude over an adjustable frequency range. This signal feeds an electric coil, which excites a magnet (NdFeB) with a mass of 0.7 g stuck to the bar so that the coil and the magnet introduce a vibration force on the bar. At another point is located a small accelerometer (Brüel & Kjaer Model 43-74) with a mass of 0.65 g. Its signal is integrated with respect to time and, accordingly, an electric signal proportional to the vibration velocity of the bar can be received. Finally, the ratio between the vibration velocity and the amplitude of random excitation versus frequency is recorded on the analyzer.

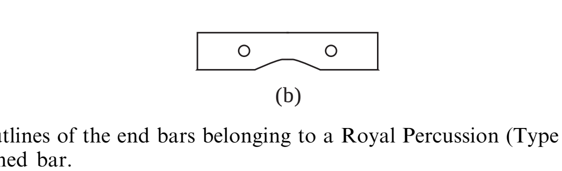
This set-up was used to make vibrational measurements of natural eigenmodes corresponding to xylophonic bars. Initially, the nodal lines for the natural eigenmodes of each bar were controlled by using Chaldni's method. In this sense, they were supported in these zones when being tested in the experimental set-up. Consequently, free-free boundary conditions were achieved.

In the vibrational measurements of xylophonic bars that we studied all the available xylophonic instruments in the Department of Percussion Instruments of the "Conservatorio Superior de Música" Bilbao (Spain). They were two marimbas ("Concorde" (Holland) and "Royal Percussion" Studio-49 (Germany)), two xylophones ("Premier" (U.K) and "Royal Percussion" Studio-49 (Germany)), a xylo-marimba ("Royal Percussion" Studio-49 (Germany)) and other smaller instruments. Nevertheless, in this work the measurements of the 44 palissandre bars comprising the "Royal Percussion" (Germany) marimba will be mostly commented on, because they represent the common behaviour of all the analyzed instruments. The musical range of this instrument includes a chromatic scale of three octaves and a perfect fifth, from F_4 ($f = 347$ Hz) to C_8 ($f = 4180$ Hz). The bars are graduated only in length (L), which ranges from 274 mm for the F_4 bar to 110 mm for C_8. The width of the bars is 34 mm and the height (h) at their ends is 20 mm. The undercut shape of these xylophonic bars varies: it is very large for the longest bars and gradually decreases in depth and length for shorter bars. A digitalized outline of the end bars of this marimba can be seen in Figure 2.

3. EXPERIMENTAL RESULTS AND DISCUSSION

Initially, the nodal lines of the natural eigenmodes for each of the 44 xylophone bars were detected. In the particular case of the fundamental eigenmode, these nodal lines are close to the location of the holes used to fix the bars to the xylophone. Accordingly, the system supporting the xylophone bars does not strongly perturb the fundamental eigenmode [6]. This eigenmode is therefore the most important one for the determination of the musical qualities of the instrument.

On making the vibrational measurements, the random responses at the centre and at one end of each xylophone bar were obtained in a range from 0 to 12 kHz together with



(a)

(b)

Figure 2. Digitalized outlines of the end bars belonging to a Royal Percussion (Type Studio-49) xylophone. (a) F_4 tuned bar; (b) C_8 tuned bar.

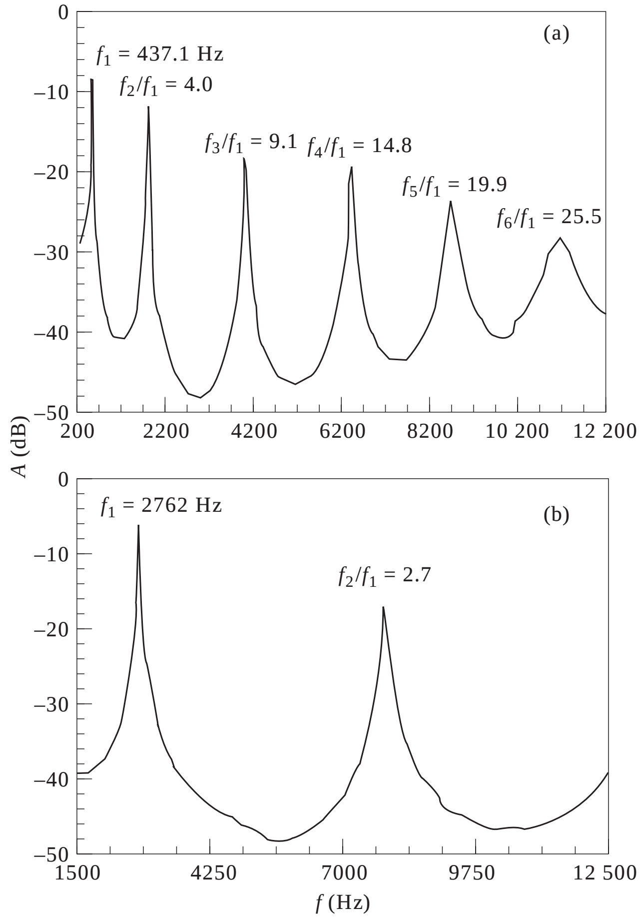


Figure 3. Response spectra of a xylophone bar to a random signal located at one end. (a) A_4 tuned bar; (b) F_7 tuned bar.

zooms of a minor frequency range for each flexural natural eigenfrequency. In Figure 3 are offered two examples of these spectra; the first represents a random response of the A_4 bar ($f = 437{\cdot}1$ Hz) in a range 0–12 kHz, while the second one is the same for the F_7 bar ($f = 2762$ Hz). These spectra were obtained by locating both magnet and accelerometer exactly at the middle of one end of the bar. Absolute values for the input admittance were achieved, but they have been plotted in a decibel scale related to a reference value of 1 m s^{-1} N^{-1}.

The maximum of each resonance is very well defined and is completely isolated from its neighbours. Moreover, since the F_7 bar is located in a range of frequencies higher than the A_4 bar, the frequency step between consecutive resonances is also higher, and this is why the isolation of each resonance maximum is considerably larger.

By using these spectra, the values of the natural frequencies, input admittance, and bandwidth corresponding to the natural flexural eigenmodes for each bar were calculated. The experimental error in the measurements was estimated to be less than 0.5% in the case of the the frequencies; 2% for the bandwidths and less than 1 dB for the input admittance.

The acoustical information obtained with these measurements can be summarized as follows.

3.1. NATURAL FREQUENCIES

The relationships between the frequencies of the first four overtones (corresponding to flexural eigenmodes) and the fundamental frequency are plotted with respect to fundamental frequency in Figure 4.

It can be observed that the integer relationships are 1:4 and that they are present only between the two first frequencies and for the 17 first bars (a chromatic scale from F_4 ($f = 347$ Hz) to A_5 ($f = 875$ Hz)). In these 17 bars the undercut is able to tune the fundamental frequency into the desired musical sound and place the first overtone two octaves higher. [7].

In the other bars (from $A_5^{\#}$ (928 Hz) to C_8 (4180 Hz)), there is no harmonic value for the $f_2:f_1$ relationship, and it decreases smoothly from the 1:4 ratio a value of 1:2·6, corresponding to the last bar (C_8). In this set of bars, the tuning of the first overtone two octaves above the fundamental (1:4 ratio) would raise the second partial up to 4 kHz. Thus, the instrument maker begins to find its tuning more difficult, especially for the highest tone bars. Similar behaviour has been encountered in the xylophonic instruments analyzed in this work.

The same line of reasoning also suffices to explain why integer relationships are not observed for partials higher than the second one. Only a 1:10 has been encountered for the third partial in bars tuned below 350 Hz. These low tone bars belong to a large marimba ("Concorde" (Holland)), and thus the 1:10 ratio has not been found in the marimba described in this work, because its range begins from 347 Hz (F_4). Nevertheless, an average value of 1:9·2 for the lowest eight bars—which decreases smoothly for the more treble ones—has been found in the latter instrument. This 1:9·2 ratio is signalled by some researchers as one of the common relationships for the third partial of marimba bars [5].

All of the frequency ratios feature the common property of a gradual decrease for the higher tone bars. Accordingly, the undercut process is useful not only for achieving harmonic relationships but is also very important for increasing the logarithmic frequency step between two consecutive resonances of each bar.

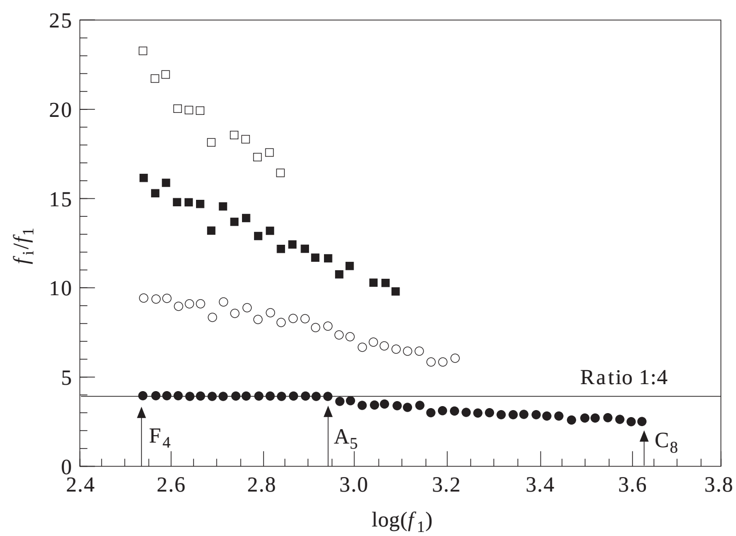


Figure 4. Values for the relationships between the frequencies of the natural bending eigenmodes of xylophone bars and the fundamental frequency shown with respect to the logarithm of the fundamental frequency of each bar. ●, f_2/f_1; ○, f_3/f_1; ■, f_4/f_1; □, f_5/f_1.

3.2. INPUT ADMITTANCES

Concerning the data obtained on input admittances, the most interesting results are the values of this magnitude for the fundamental eigenmode measured at the centre and at one end of each xylophonic bar. As a particular case in Figure 5 is shown the input admittance of the first eigenmode at the centre of each bar versus the logarithm of its fundamental frequency. For these measurements, both the accelerometer and magnet were located in the middle of the central line of the bar. As in the case of the spectra corresponding to Figure 3, the absolute values of the input admittance are presented in the decibel scale related to the reference of 1 m s^{-1} N^{-1}.

A decrease in input admittance can be seen for higher frequency bars. This kind of behaviour is typical not only of xylophones but also of any kind of musical instrument: the higher the frequency tones desired, the smaller are the geometrical parameters of the vibrating element. Consequently, short bars are unable to reach vibrational amplitudes as great as those corresponding to low tone bars. In xylophones it is specially important that this unavoidable decrease in amplitude should exhibit smooth behaviour over the musical range of the instrument. Nevertheless, certain sharp changes in input admittance between consecutive bars often occur in xylophones, although, finally, they are mitigated by the effect of the resonators (metal tubes).

The effect of undercut shape is noticeable in the equal or higher values shown by the input admittance at the centre of each bar than at its ends [7]. Thus, the undercut process improves the pattern of vibration of xylophonic bars in order to respond more strongly to attack by the mallet and to be more efficient as a radiator. Moreover, the increase in vibrational amplitude at the centre of the bar is always desirable because the mallet is often used to beat the bar from its ends towards its centre to achieve expressive effects, such as in crescendo.

Finally, with regard to the measured values of input admittances for overtones, these are much smaller than those corresponding to the fundamental. This does not mean however, that the role of overtones can be dismissed as negligible; rather, by using

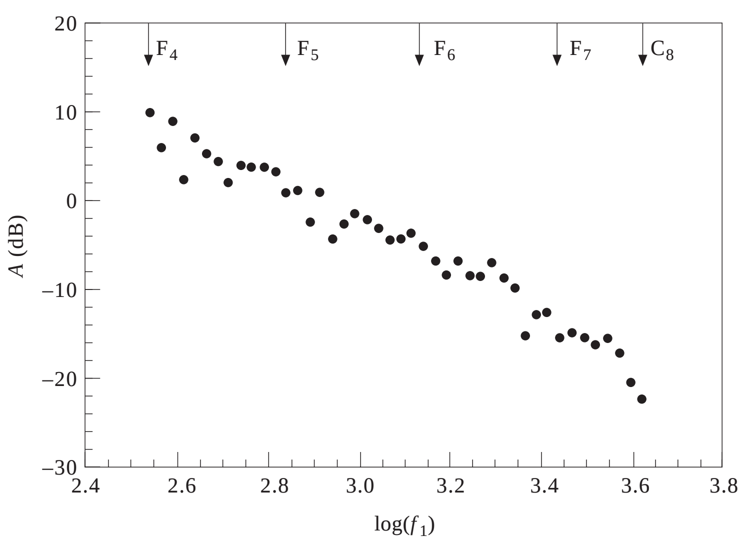


Figure 5. Values for the input admittance of the first eigenmode at the centres of xylophone bars shown versus the logarithms of their fundamental frequencies.

hard-headed mallets and hitting with them at certain points, overtones can be very strongly reinforced.

3.3. DAMPING

If one wished to study the damping of the sound provided by a musical instrument properly, it would be necessary to take into account the mechanical coupling among the vibrating elements of the instruments and its resonator elements. Measurements of the output sound generated by the musical instruments would be required for such a purpose. However, in this work, only the generating of this sound at the bars of the xylophone has been studied.

Measurements have consisted only of the bandwidth of each resonance. From these data the decay time can be calculated. This magnitude has been finally used because it is related to the damping more closely than bandwidths. In Figure 6 is given a plot of the decay time for the two first flexural eigenmodes of each xylophone bar against the fundamental eigenfrequency. It can be observed that the decay time decreases for higher frequency bars; in other words, the decay of the resonances corresponding to these bars is faster. The reason for this is similar to that given to explain the decrease in input admittance: the higher the frequency desired, the smaller the vibrating element used. Consequently, the decay of the oscillations will be faster. With respect to this increase in damping versus the first frequency of the bar, in order to obtain a smooth increase in damping from low to high frequency bars it is very important that there should be no sharp steps between consecutive bars. This is the general kind of behaviour shown in Figure 6.

It can also be stated that the decay times for the first overtones are lower than those corresponding to the fundamental frequency. This is why in the output sound produced by the xylophone the overtones tend to vanish before the fundamental.

By using the data obtained for the bandwidths, the quality factor (Q) for the fundamental resonance of each bar can also be calculated. For the palissandre bars of the xylophone analyzed here, the average of Q is 200 for the fundamental frequency. This is a very high Q value in comparison with the values obtained for other kinds of wood. It means that palissandre is highly suitable for avoiding a sharp damping of the sound after

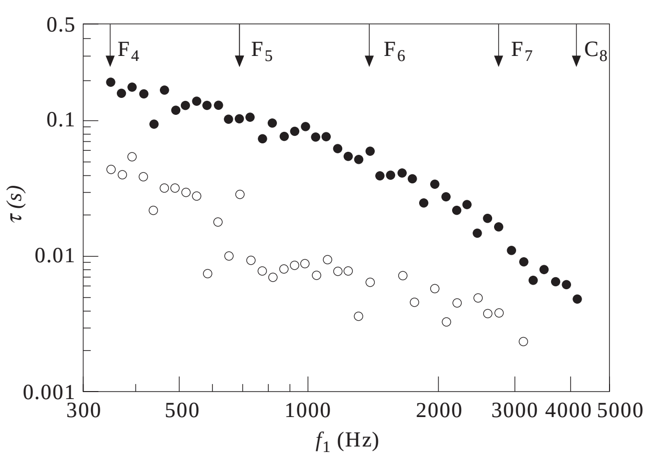


Figure 6. Values for the logarithm of the decay time corresponding to the two first natural bending eigenmodes corresponding to xylophone bars versus the logarithms of their fundamental frequencies. ●, Decay time for the first eigenmode; ○, Decay time for the second eigenmode.

impact by the mallet, which is well withstood by the bar owing to the high density and stiffness of palissandre wood [9] ($\rho = 1050$ kg/m^3, $E = 19$ GPa).

3.4. TUNING SYSTEM

Finally, by taking the above results into account the tuning system used in this musical instrument can be studied. In most cases, xylophones are tuned to a tempered scale and their bars are situated on two levels, the higher one for altered notes and the lower one for natural nodes. Nevertheless, in Chinese and old African xylophones the bars are located on only one level and are usually tuned to a pentatonic scale. The study of the tuning system used in the instrument examined here will not be quite exact, because an approximation will be made: pitch quality will be related only to the periodicity of the fundamental resonance, and the influence of overtones will be neglected. However, this is quite a good approximation, because the overtones are located far away enough in frequency with regard to the fundamental eigenmode [10]. With this approximation, it can be observed that the tuning system is very close to the equal temperament, because the strongest deviations registered are only of about 10% close to the frequency discrimination of the human ear [10].

4. CONCLUSIONS

The main aim of this work has been to offer a general description of the physico-acoustical behaviour of xylophonic instrument bars. For this objective, not only the natural frequencies, but also the vibrational amplitudes and natural damping of the bars have been measured. With this information it has been possible to gain a deeper insight into the effects caused by the undercutting process on the vibrational behaviour of xylophone bars. Thus, this process should not only be considered as a way in which to achieve harmonic relationships between frequencies but also as a way in which to control the relationships among all frequencies (not just the fundamental ones), and to improve the pattern of vibrations corresponding to xylophonic bars.

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